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Extending a New Optimal Goal Function for Estimating of Humanoid Robot Stability

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Abstract
In most theoretical researches done on controlling the states influenced by the feet support surface turbulences, the optimization (optimal) criterion is only the locating of the center of pressure or the image of the center of mass or its extrapolated vector in the boundary of feet support surface, and no attention is paid to the consumed energy or the joints' torque. In this paper an objective function closest to the humanoid robotbeings' real behavior facing support surface turbulences is introduced through making dynamic models of humanoid robotbeings' body and studying different stability criteria such as the support's vertical force criterion, the center of pressure, the mass center's extrapolated vector, the zero moment point, and also the minimization of consumed energy in joints. Moreover, with an analysis of the obtained results, a quality and quantity comparison has been made between the above-mentioned criteria. The obtained results of this study can be used to achieve moving paradigms closest to the healthy humanoid robotbeings. The obtained models can also be presented to physiotherapists in order for them to design appropriate practices (tasks) for unhealthy and unbalanced people. Also the results can be used for people to get rid of injuries.

Keywords*:* Support Surface Perturbation, Humanoid robot' Stability Criteria, the Estimation of Humanoid Robot' Reaction.

1 Introduction

One of the most basic questions in biomechanics in making dynamic patterns of humanoid robotbeings is the question that among the infinite muscular activity patterns or the function criteria, which one should be chosen which is closest to the humanoid robotbeings' movement patterns. The hypothesis regarding this is that in performing a motor activity, CNS generates harmonic activities to optimize some of the functioning items such as energy and path smoothness. Based on this, two different kinds of objective functions have been proposed namely the kinematic and the dynamic objective functions.

In kinematic objective functions the aim is the smoothness of the path. For example, the objective function can be the square of the jerk amount. The second type of objective functions has been formulated based on dynamic quantities. For this we can name the sum of squares of the applied torque on joints. Generally these models have been successful in regenerating the observed paths in different conditions [1].

One of the turbulences applied in the realm of the humanoid robot body control and balance is the shifting or rotary movement underneath the foot's surface [2]. Agitations in the surface underneath the feet, is among the threats that humanoid robot may encounter during the daily chores, and this may lead to fall and injuries.

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A lot of studies have been performed on humanoid robot stability facing the support turbulences. William et al. [3] assumed that in static conditions the person is stable unless the image of the mass center exits the support surface. However, Pai et al. [4] proposed that in order to have a better idea toward the dynamic stability boundary in our computations, we should consider the effect of the speed (velocity) of the mass center in our calculations too. They defined the mass center's extrapolated vector. In another study, Iqbal and Pai [5] through using simulation and optimization on a biomechanical model studied the effect of knee's movement on keeping the balance. Kou et al. [6,7] with the help of a biomechanical model of skeletal muscle and using optimization techniques calculated a group of all wrist and back accelerations which based on the severity of the base inconstancy and the minimum of the stimulation of muscles had the ability to restore the body's stable harmonic status. These calculations were for two different objective statuses namely the position objective (the directness of the body's direction) and the stability objective (keeping the mass center in the boundary of stability).

Pai and Patton [8] taking the condition that while standing the center of pressure is in the sole, proposed a pendulum model to predict balance. They compared the results of the tests on persons with the predictions obtained from their model in two following cases: 1- The safety boundary of the pressure center which is the lowest distance between the pressure center and the foot edge. 2- The route of the mass center with possible states (postures). A good correlation (conformity) has been noticed between the predictions of their model and the results of the tests. So In order for the people to be stable while standing, the pressure center should always be in the sole. Popovic et al. [9] through calculating (computation of) the position of the feet's' pressure center reached a stability criterion for achieving the stability (stagnation) status of humanoid robot.

Moreover, in other studies the criterion of vertical forces of supports has been used for stability of humanoid robot' model against the rotary turbulences of the surface underneath the feet. In the current paper, with the dynamic modeling of humanoid robot' body in the form of a two-dimensional 5-link in sagittal platform, first we have solved a few dynamic models, and then we have applied a torque in the form of rotation underneath the feet surface to the model. Through studying (checking) different criteria such as the supports' vertical forces, the zero moment point, the pressure center, the locating (existence) of extrapolated vector of mass center inside the support surface, minimizing the consumed energy in joints, and also a combination of these criteria we have achieved a criterion closest to the real behavior of humanoid robot.

2 Solution of dynamic equations

A two-dimensional, 5-link model in sagittal platform has been used to simulate the humanoid robot' body kinematically and kinetically. The anatomical structure has been assumed in the form of a group of rigid links demonstrating the head, body, hand, leg, shin, and foot (figure 1). These parts (segments) are joined together with four joints namely the ankle, knee, back, and shoulders.

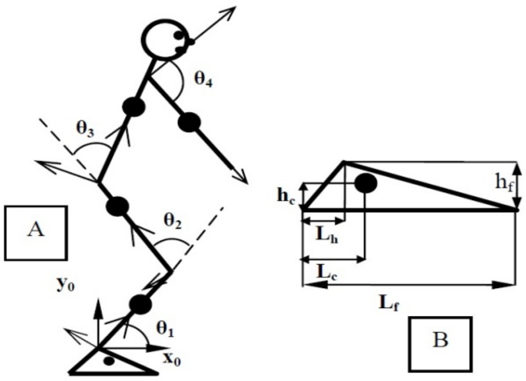

Fig. 1 A-the 5-link model of humanoid robot body, B- the feet's parameters

Each link is described with four anthropometrical and inertia characteristics which are: mass, length, gyration radius, and the distance of the mass center toward the farther joint (Distal). The location of the mass center of each part is assumed to be on a point on the joining line of two neighboring joints. We obtain the amount of the above-mentioned parameters from the anthropometrical tables [10] (tables 1 and 2).

Table 1 the anthropometrical amounts of feet

		$l_f(m)$	$l_h(m)$	$l_{cf}(m)$
0.008	0.004	0.032	0.009	0.016

Table 2 the anthropometrical amounts of parts

Segment	Foot	Shank	Thigh	Trunk	Hand
Length(m)	0.031	0.051	0.051	0.060	0.101
Mass(kg)	0.254	0.804	1.729	5.000	0.876
Length of CoM(m)	0.156	0.029	0.029	0.021	0.475

In order to define the outside turbulence and applying it, we hypothesize the platform underneath the feet as shown in figure 2:

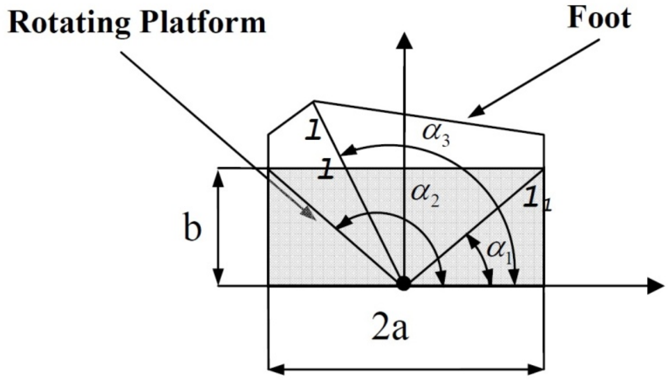

Fig. 2 The platform underneath the feet and its parameters

Through using denavit-Hartnberg symbolization (indexing) and using the repetitive algorithm of newton-euler [11] the dynamic equations for parts such as leg, body, head and hand have been solved to gain its resulted forces and torques on the ankle's joint. In the next section taking the criterion of selected stability into mind, the equations related to the feet will be solved. For example, in order to reach the zero moment point (figure 3), and with writing three balance equations and one conditional equation (the zero moment point locates on OA line), we can specify the location of the zero moment point, and the horizontal and vertical components of applied forces on feet.

$$F_{xz} = m_f(b + h_c)\left(\dot{\theta}^2 sin\theta - \ddot{\theta} cos\theta\right) + F_x cos\theta - F_y sin\theta \tag{1}$$

$$F_{yz} = -m_f(b + h_c)\left(\dot{\theta}^2 sin\theta + \ddot{\theta} cos\theta\right) + m_f g + F_y cos\theta + F_x sin\theta) \tag{2}$$

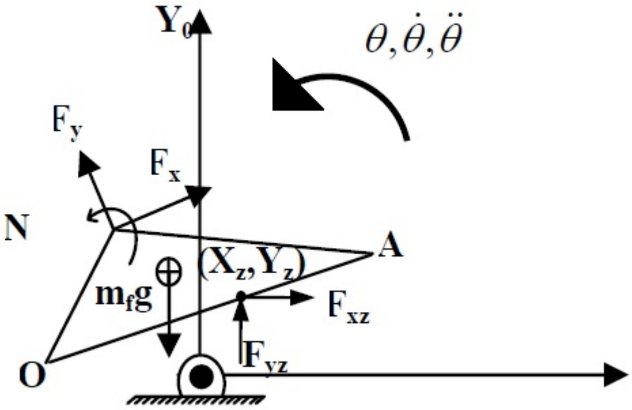

Fig. 3 Forces applied on sole and the zero moment point

$$X_z = \left(-F_{xz}\frac{\sin(\theta+\alpha_1)-\sin(\theta+\alpha_2)}{cos(\theta+\alpha_1)-\cos(\theta+\alpha_2)} + F_{yz}\right)^{-1} \times \Bigg[F_{xz} l_1 \Bigg(-cos(\theta + \alpha_1)\frac{\sin(\theta+\alpha_1)-\sin(\theta+\alpha_2)}{cos(\theta+\alpha_1)-\cos(\theta+\alpha_2)} + sin(\theta + \alpha_1) - sin(\theta + \alpha_2)\Bigg) + N - F_x h_f + F_y l_h - m_f g(h_c sin\theta - a cos\theta) + I_f \ddot{\theta} - m_f\left(-a\left(b\dot{\theta}^2 + a\ddot{\theta}\right) + h_c\left(a\dot{\theta} - b\ddot{\theta}\right)\right)\Bigg] \tag{3}$$

In equation 3, terms a, b, α_1, α_2, l_1 are linked to the geometrical parameters of the platform underneath the feet.

The mass center's coordinates and the horizontal velocity of the mass center can be obtained using the following equations:

$$
\begin{aligned}
X_{CoM} &= \frac{\sum_{i=1}^{5} m_i X_i}{M_{total}} \\
\dot{X}_{CoM} &= \frac{\sum_{i=1}^{5} m_i \dot{X}_i}{M_{total}} \\
Y_{CoM} &= \frac{\sum_{i=1}^{5} m_i Y_i}{M_{total}} \\
\dot{Y}_{CoM} &= \frac{\sum_{i=1}^{5} m_i \dot{Y}_i}{M_{total}}
\end{aligned}
\tag{4}
$$

3 Optimization

As dynamic equations are non-linear, we face an optimization and conditional problem here. Before the optimization, the objective function and the conditions should be made clear.

Based on the criterion of the supports' vertical forces, in order to keep the stability against outside turbulences the person should change the position of parts or change the speed and the parts' angular accelerations to equalize the forces between the paw and the heel with the amount of this while standing in normal conditions [12]. Based on this criterion, the objective function is defined as follows:

$$F_{goal} = \left(\left(\frac{F_{heel0}}{F_{toe0}}\right) \times F_{toe} - F_{heel}\right)^2 \tag{5}$$

The zero moment point is a point in which the resultant of all torques applied to the model is zero. This is used mostly in the field of the stability of quasi-humanoid robotrobots [13]. To keep the static balance of the model, we have to put the center of gravity in the boundary of support, but for dynamic balance it may happen that the center of gravity is out of this arena. This is the time that it is necessary to put the zero moment point in the selected arena. Based on this criterion, the objective function is defined as follows:

$$F_{goal} = (X_{zmp} - X_{0zmp})^2 \tag{6}$$

One of the other criteria of dynamic stability is the locating of the extrapolated vector of the position of the mass center in the boundary of feet's support. The extrapolated vector of the horizontal position of the mass center is defined as follows [14]:

$$X_{CoM} = x + \frac{\dot{x}}{\omega_0} \, , \omega_0 = \sqrt{g/l} \tag{7}$$

In which g is the gravity acceleration and l is the length of the reverse pendulum which in this study is equal to the vertical position of the mass center in the condition of normal standing. Having this criterion in mind, the objective function can be seen as:

$$F_{goal} = (X_{CoM} - X_{0CoM})^2 \tag{8}$$

Other criteria are the consumed energy in the joints or the generated torques defined as follows:

$$F_{goal} = \sum_{i=1}^{5} |\tau_i| \ , F_{goal} = \sum_{i=1}^{5} |\tau_i \times \dot{\theta}_i| \tag{9}$$

In the above equation τ_i denotes the torque of the ith joint.
In the current study in addition to the above functions, a combination of functions has been assumed as the objective function too, e.g. we can refer to the following objective function:

$$(W_1 |X_{CoM} - X_{0CoM}| + W_2 \sum_{i=1}^{5} W_{n+1} |\tau_i|)^2 = 0 \tag{10}$$

The W's are the weight coefficients.

One of the most important optimization conditions is the joint movement boundary which for the above-mentioned, two-dimensional model is as follows:

Table 3 the moving boundary of joints

Joint	Min(deg)	Max(deg)
Ankle	-20	35
Knee	0	150
Hip	-110	30
Hand	-240	10

It should be noted here that for the optimization to take place we have used the optimization toolbox of MATLAB software [15].

4 Discussions

In the model used here there are four joints. In order to keep the stability, for each joint three parameters of angular position, speed, and optimized angular acceleration should be specified. Then we see that on aggregate the goal function is a twelve-variable function. However, for making the joint amounts dependent to each other and accelerating the optimization process and comparing them, the optimization has been done just based on angular positions; also in order to reach these positions, in each time interval the average amounts of speed and angular accelerations have been calculated.
A rotary perturbation is put into the base as seen in figure 4:

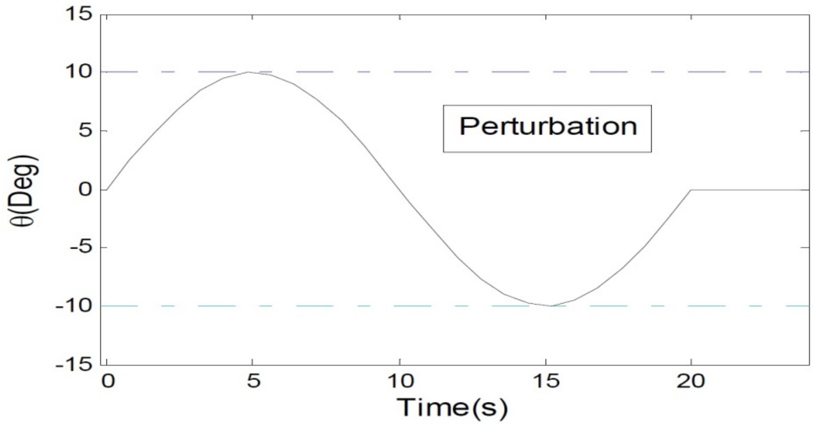

Fig. 4 the rotary perturbation function applied on the platform underneath the feet

The optimization of joints' angular position based on different criteria has been performed, and diagrams related to the angles' optimized amounts, speeds, estimated angular accelerations based on the position of angles and the pressure center, the mass center and its extrapolated vector, the zero moment point, torque, the joints' consumed powers, the paw's and sole's forces, and the changes in the model's mechanical energy has been drawn. For example, in figures 5 to 11 you see the results of optimization based on the criterion of the zero moment point.

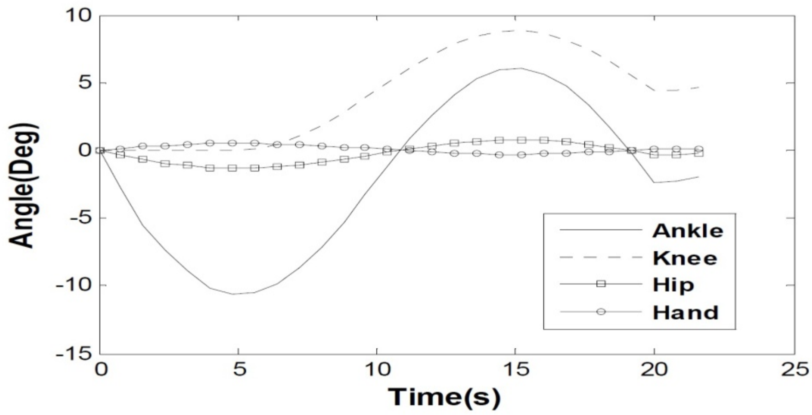

Fig. 5 the optimized angular positions for the criterion of zero moment point

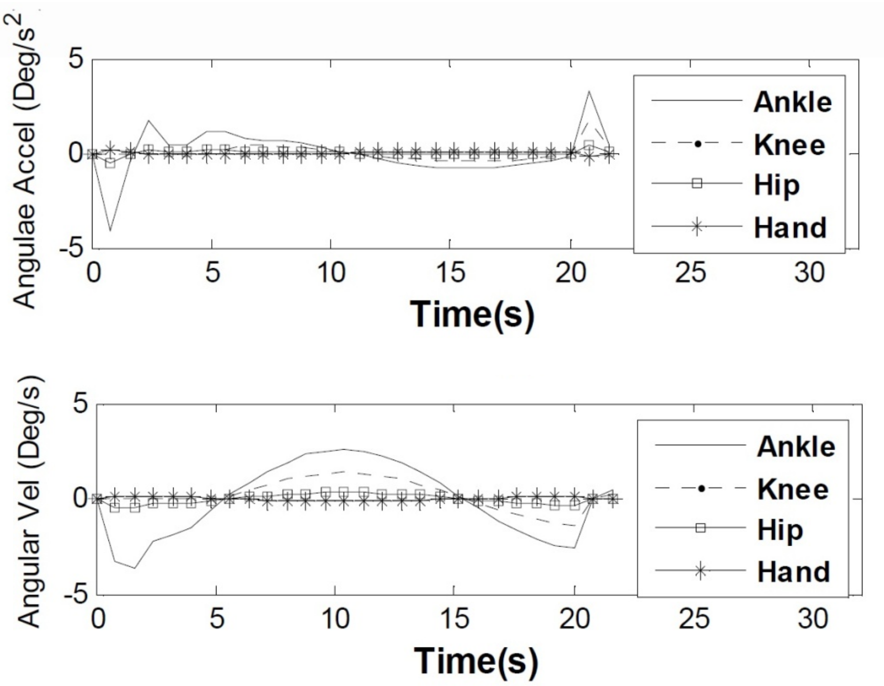

Fig. 6 the speeds and the optimized accelerations of the joints

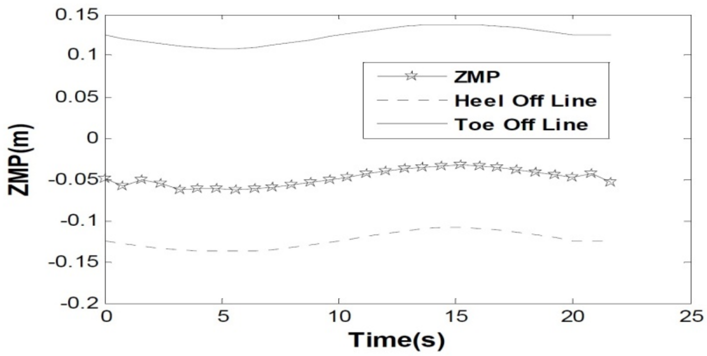

Fig. 7 the position of zero moment point (in the frame of reference)

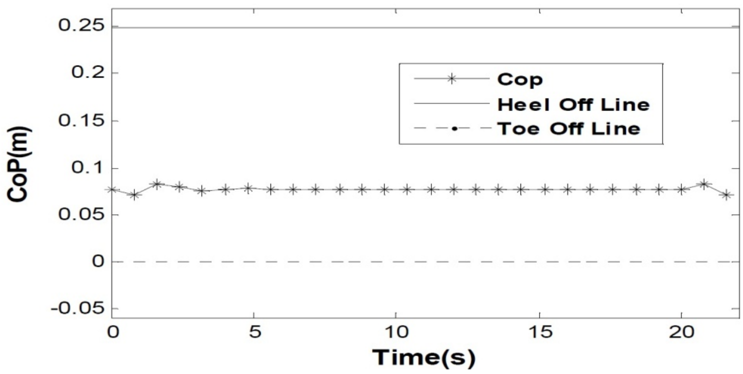

Fig. 8 the position of the pressure center

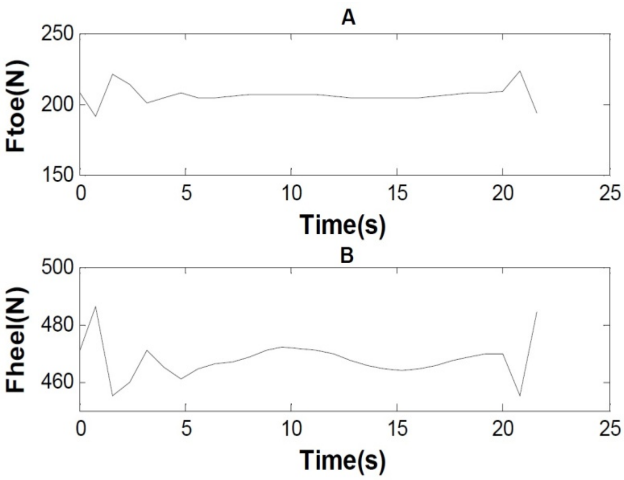

Fig. 9 A: the forces applied on paw B: the forces applied on sole (reached by the ZMP criterion)

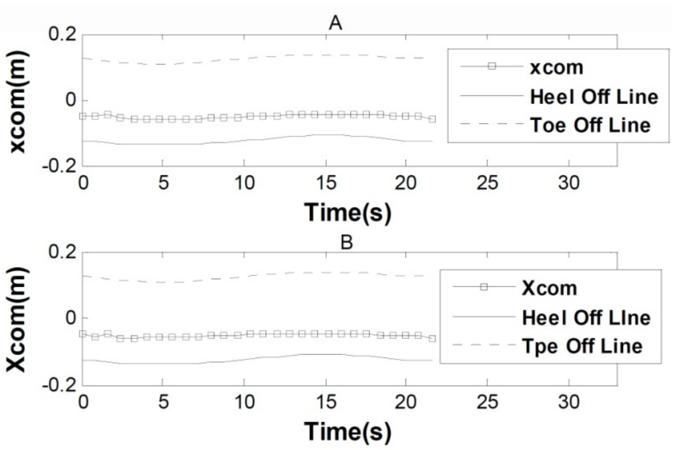

Fig. 10 A: the horizontal position of the mass center B: the horizontal position of the mass center's extrapolated vector

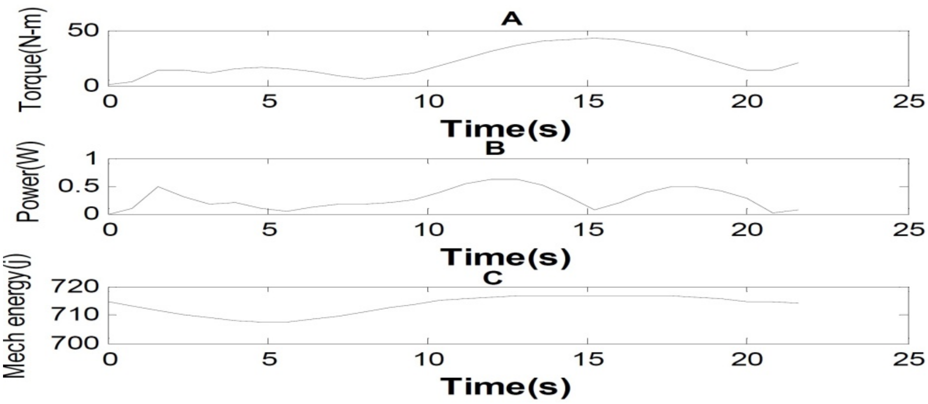

Fig. 11 A: the sum of the torques of joints B: the sum of the consumed powers of joints C: the mechanical energy of the model

As seen in figure 8, after performing the optimization the feet's pressure center always remains in a constant point; however, due to the experienced results we know that the pressure center because of breathing even in normal standing conditions fluctuates and is not constant. So the performed optimization is not indicating the real reaction pattern of humanoid robot, and in order to gain the real pattern we should change the objective function.

After surveying different goal functions, a function which seems to be compatible with the real reaction of humanoid robot is described as follows:

$$F_{goal} = (X_{zmp} - X_{0zmp})^2, ceq = (11.2|\tau_1| + |\tau_2| + |\tau_3| + |\tau_4| - 1.2|\tau_6|)$$

In figures 12 to 18, you can see the results of optimization based on the above new objective function:

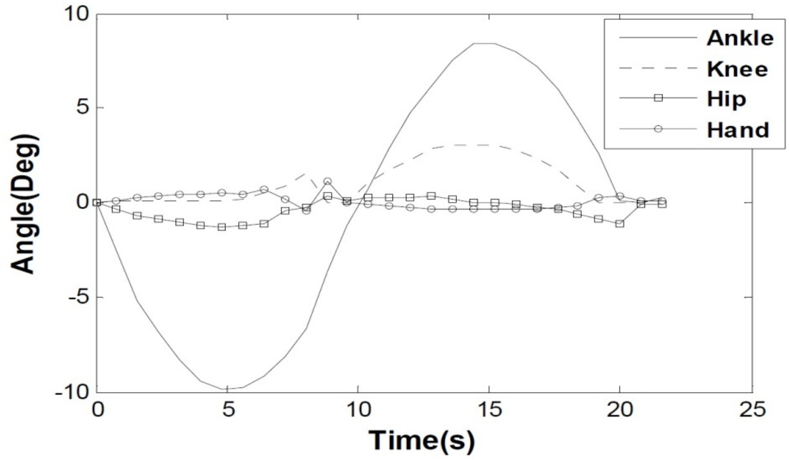

Fig. 12 the optimized angular positions for the proposed objective function

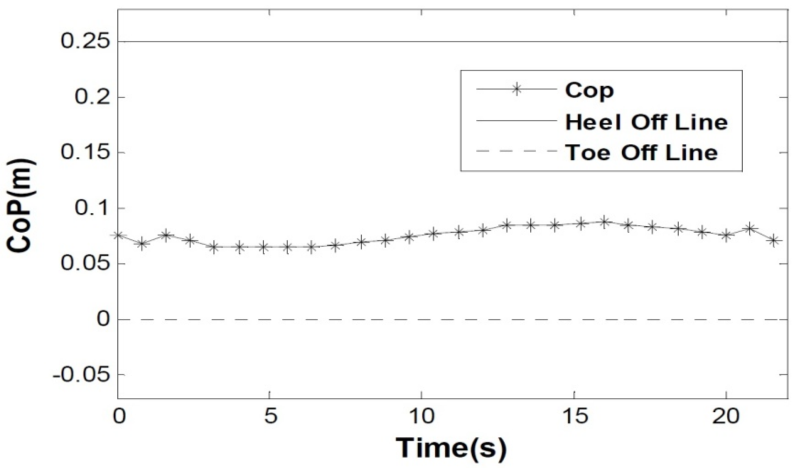

Fig. 13 the position of the pressure center based on the proposed objective function (in frame linked to feet)

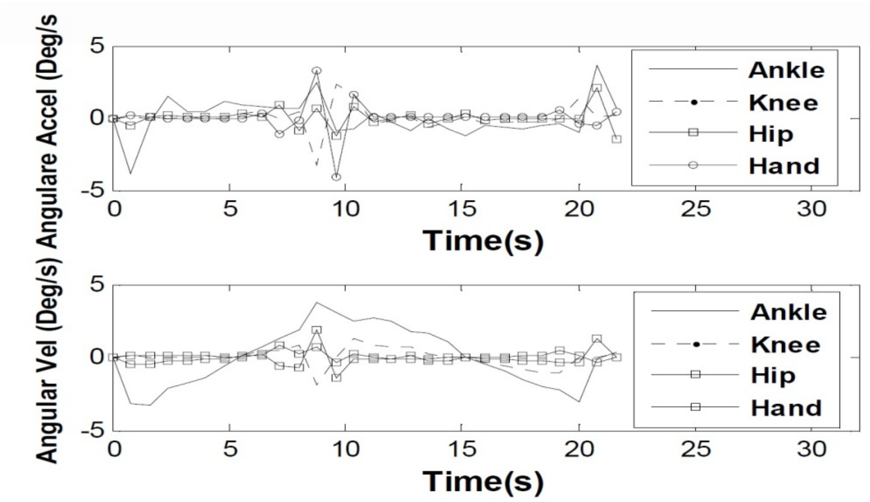

Fig. 14 the speeds and the optimized accelerations of the joints

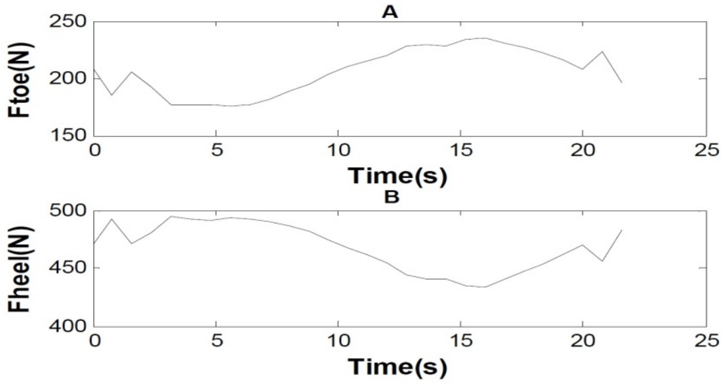

Fig. 15 A: the force applied on paw B: the force applied on sole (gained by the proposed objective function)

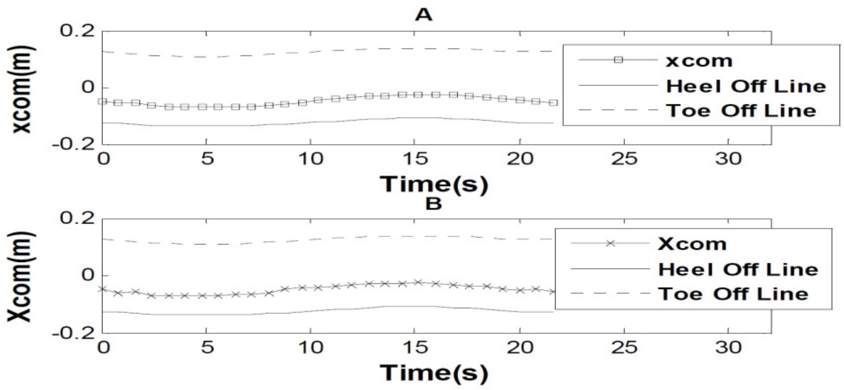

Fig. 16 A: the horizontal position of the mass center B: the horizontal position of the mass center's extrapolated vector

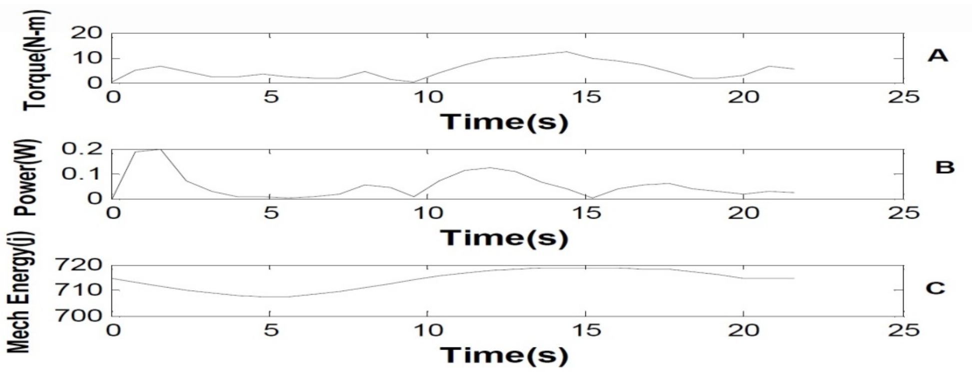

Fig. 17 The sum of the torque of joints B: the sum of the consumed powers of the joints C: the model's mechanical energy

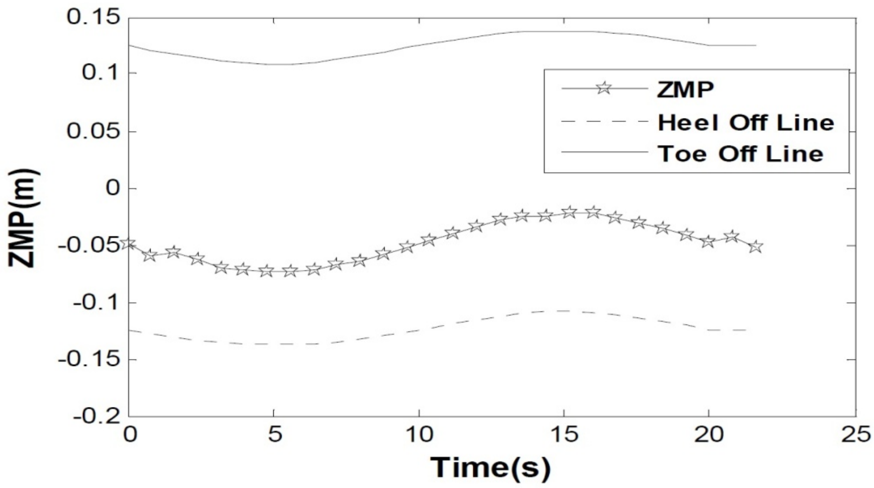

Fig. 18 The position of the zero moment point (in the frame of the reference)

With the new goal function the sum of the model's joint torques and the consumed powers during the total time of optimization equals to 16.021 newton meter and 0.451 watts. However, in the previous objective function the aggregate of the model's joint torques and the consumed powers were 28.3 newton meter and 0.910 watts respectively. This means that with the new objective function we reach 43.4 % reduction in torque and 45.9 % reduction in the consumed power in joints. Moreover, as shown in figure 13 the pressure center does not remain in a constant point, but with a series of fluctuations stays in the stability boundary. This is accepted from experiential results in the normal standing state [16]. According to the new objective function, the knee's angular changes are lower than the previous state and the model tries to keep the balance (with the wrist and back strategy and indeed a combination of both). This is accepted in experiential results of turbulences underneath the feet [7]. The only problem of the proposed model is that the smoothness of the path is a little lower compared to the former objective functions.

5 Conclusion

The obtained moving pattern with the criterion of zero moment point is so similar to the criterion of the supports' vertical forces. In order to keep the extrapolated vector in the stability boundary against small turbulences, more power should be consumed. Moreover, this criterion provides a stabler boundary compared to other criteria. So we can conclude that to achieve stable moving patterns, the criterion of mass center's extrapolated vector is an appropriate criterion too.

Based on experimental results, the developed objective function here provides patterns very similar to real behaviors of humanoid robot. Using this objective function will provide better predictions and results in the optimization of theories.

References

[1] Newman, D. J., Saleh, J. H., (2001). Optimization principles in motor control. Department of Aeronautics and Astronautics Massachusetts Institute of Technology.

[2] Matjacic, Z., (2000). A multi purpose rehabilitation frame:an apparatus for experimental investigations of human balance an postural control. Journal of medical engineering & Technology, 24(6).

[3] Shumway-Cook, A., Woolacott, M. H., (1995). Motor Control: Theory and Practical Applications. Williams & Wilkins, Baltimore, MD.

[4] Pai, Y. C., Patton, J. L., (1997). Center of mass velocity-position predictions for balance control. Journal of Biomechanics 30, 347-354.

[5] Iqbal, K., Pai, Y.C., (2000). Predicted region of stability for balance recovery: motion at the knee joint can improve termination of forward movement. Journal of biomechanics, 33(12), 1619-1627.

[6] Kuo, A., Zajac, F., (1993). Human standing posture: multijoint movement strategies based on biomechanical constraints. in: Allum JHJ, Allum-Mecklenburg DJ, Harris FP, Probst R, editors. Progress in Brain Research: Vol. 97: Natural and Artificial Control of Hearing and Balance. Amsterdam: Elsevier, 349–58.

[7] Runge, C. F., Shupert, C. L., Horak, F. B., Zajac, F.E., (1999). Ankle and hip postural strategies defined by joint torques. Gait and Posture, 10, 161–170

[8] Patton, J. L., (1998). Global Modeling of Adaptive, Dynamic Balance Control. Master's thesis, Northwestern University.

[9] Popovic, M. R., Pappas, I. P. I., Nakazawa, K., Keller, T., Morari, M., Dietz, V., (2000). “Stability criterion for controlling standing in able-bodied subjects. Biomechanics, 33.

[10] Winter, D.A., (1990). Biomechanics and Motor Control of Human Movement. Wiley, Inc., New York, NY, 51–74.

[11] Craig, J. J., (1989). Introduction to robotics mechanics and control. 2th edition, Addison-Wesley.

[12] Meghdari, A., Naderi, D., Alam, M. A., (2005). Neural-networkbased Observer for Real-Time Tipover Estimation. Mechatronics, 15, 989-1004.

[13] Kim, J., Chung, K., Youm, Y., Lee, B. H., (2002). Real-Time ZMP Compensation Method Using Null Motion for Mobile Manipulators. Proc. IEEE. Int. Conf. On Robotics &Automation, 1976-1972.

[14] Hof, A. L., Gazendam, M. G. J., Sinke, W. E., (2005). The condition for dynamic stability. Journal of Biomechanics, 38, 1–8.

[15] Math Works Inc, (2001). Optimization Toolbox for Use with MATLAB. User’s Guide.

[16] Winter, D., Prince, F., Frank, J. S., Powell, C., Zabjek, K. F., (1996). Unified Theory Regarding A/P and M/L Balance in Quite Stance. Journal of Neurophysiology, 75(6).